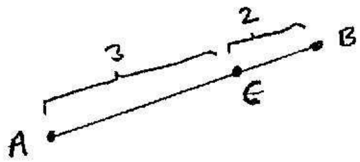
P 72

[1.1]



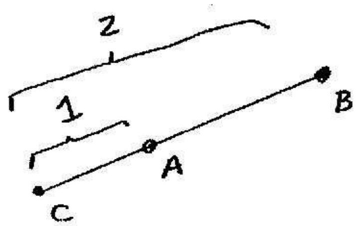
$$\vec{AC} = \frac{3}{5}\vec{AB}$$

$$\vec{c} - \vec{a} = \frac{3}{5}(\vec{b} - \vec{a})$$

$$\vec{c} = \frac{3}{5}\vec{b} - \frac{3}{5}\vec{a} + \vec{a}$$

$$\vec{c} = \frac{2}{5}\vec{a} + \frac{3}{5}\vec{b}$$

[1.2]

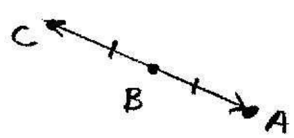


$$\vec{CA} = \vec{AB}$$

$$\vec{a} - \vec{c} = \vec{b} - \vec{a}$$

$$\vec{c} = 2\vec{a} - \vec{b}$$

[1.3]



$$\vec{BC} = -\vec{BA}$$

$$\vec{c} - \vec{b} = \vec{b} - \vec{a}$$

$$\vec{c} = 2\vec{b} - \vec{a}$$

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[2.1]

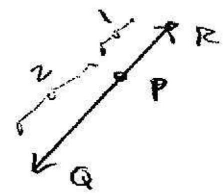
$$\vec{PQ} = \vec{q} - \vec{p} = -6\vec{a} + 6\vec{b} - 2\vec{a} - 2\vec{b} = -8\vec{a} + 4\vec{b}$$

$$\vec{PR} = \vec{r} - \vec{p} = 6\vec{a} - 2\vec{a} - 2\vec{b} = 4\vec{a} - 2\vec{b}$$

[2] Since $\vec{PQ} = -4(2\vec{a} - \vec{b})$

and $\vec{PR} = 2(2\vec{a} - \vec{b})$,

$$\vec{PQ} = -2\,\vec{PR}.$$



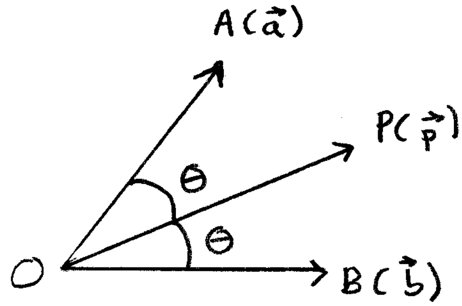
$\therefore$ Points P, Q, R colinear

In fact, P internally divides QR in ratio 2:1

[3.1]

$\vec{a} = \overrightarrow{OA}$, $\vec{b} = \overrightarrow{OB}$, $\vec{p} = \overrightarrow{OP}$.

$\vec{p}$ bisects angle AOB.



then

$$\cos\theta = \frac{\vec{a}\cdot\vec{p}}{|\vec{a}||\vec{p}|} = \frac{\vec{b}\cdot\vec{p}}{|\vec{b}||\vec{p}|}$$

So $$\frac{\vec{a}\cdot\vec{p}}{|\vec{p}|} = \frac{\vec{b}\cdot\vec{p}}{|\vec{p}|}, \quad \because\ |\vec{a}| = 1 = |\vec{b}|$$

$$\therefore \quad \vec{a}\cdot\vec{p} = \vec{b}\cdot\vec{p}$$

[3.2] as above, then

$$\frac{\vec{a}\cdot\vec{p}}{2} = \frac{\vec{b}\cdot\vec{p}}{3}$$

$$\therefore \quad \vec{a}\cdot\vec{p} = \tfrac{2}{3}(\vec{b}\cdot\vec{p})$$

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ANSWER

[4.1]

$$\boxed{\begin{array}{ll} \ell_1 : & \vec{p} = \langle 1,1 \rangle + s \langle 1,2 \rangle \\ \ell_2 : & \vec{p} = \langle 1,5 \rangle + t \langle 3,-4 \rangle \end{array}}$$

OR

$$\boxed{\ell_1 \begin{cases} x = 1 + s \\ y = 1 + 2s \end{cases} \qquad \left. \begin{array}{l} x = 1 + 3t \\ y = 5 - 4t \end{array} \right\} \ell_2}$$

IS OK, too.

* In question 11 ... "taking components of ℓ_1 and ℓ_2 as the parameters s and t". I have no idea what this might mean.

[4.2]

$$\begin{bmatrix} 1 + s = 1 + 3t \\ 1 + 2s = 5 - 4t \end{bmatrix} \sim \begin{bmatrix} s - 3t = 0 \\ 2s + 4t = 4 \end{bmatrix} \sim \begin{bmatrix} 2s - 6t = 0 \\ 2s + 4t = 4 \end{bmatrix}$$

$$\Rightarrow \quad 10t = 4 \qquad\qquad s - 3\left(\tfrac{2}{5}\right) = 0$$

$$\boxed{t = \frac{2}{5}} \qquad\qquad \boxed{s = \frac{6}{5}}$$

Then, to get coords of pt of intersection

$$x = 1 + s = 1 + \frac{6}{5} = \frac{11}{5}$$

$$y = 1 + 2s = 1 + \frac{12}{5} = \frac{17}{5}$$

$\therefore$ ℓ_1 and ℓ_2 intersect at the Point $\left(\frac{11}{5}, \frac{17}{5}\right)$

[5.1] $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{AD}$

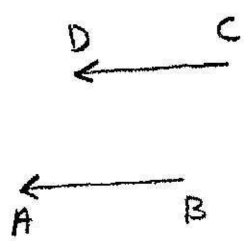
$$\Leftrightarrow \vec{c} - \vec{a} + \vec{d} - \vec{b} = 2(\vec{d} - \vec{a})$$

$$\Leftrightarrow \vec{c} - \vec{a} + \vec{d} - \vec{b} = 2\vec{d} - 2\vec{a}$$

$$\Leftrightarrow \vec{a} - \vec{b} + \vec{c} - \vec{d} = 0$$

$$\Leftrightarrow \vec{a} - \vec{b} = \vec{d} - \vec{c}$$

$$\Leftrightarrow \overrightarrow{BA} = \overrightarrow{CD}$$



$\therefore$ since opposite sides are parallel and equal, ABCD is parallelogram

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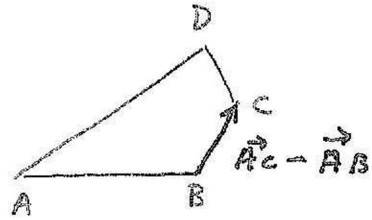
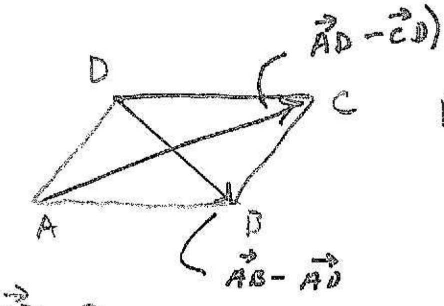
[5.2]

a) $\vec{AC} - \vec{AB} = \vec{BC}$, since
$\vec{AD} = \vec{BC}$, Figure is parallelogram.





Parallelogram

b) $(\vec{AB} - \vec{AD})$ and $(\vec{AD} - \vec{CD})$
are the diagonals
of the parallelogram.
Since the dot product
is zero, diagonals
perpendicular.

$$\left.\begin{aligned}&\vec{AD} - \vec{CD}\\ &= \vec{AD} + \vec{DC}\end{aligned}\right\}$$ Think this

$\therefore$ ABCD is a rhombus.